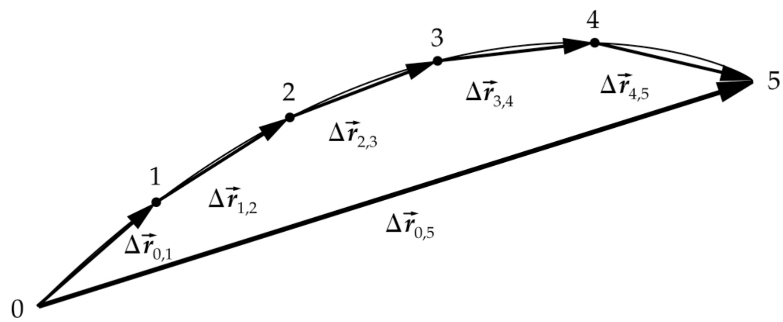
Physics 200

Chapter 3

PROBLEMS: 1, 6, 19, 23, 24, 45, 48, 53, 56, 62, 69, 72, 75, 88, 106, 111

1 • **[SSM]** Can the magnitude of the displacement of a particle be less than the distance traveled by the particle along its path? Can its magnitude be more than the distance traveled? Explain.

Determine the Concept The distance traveled along a path can be represented as a sequence of displacements.



Suppose we take a trip along some path and consider the trip as a sequence of many very small displacements. The net displacement is the vector sum of the very small displacements, and the total distance traveled is the sum of the magnitudes of the very small displacements. That is,

$$\textit{total distance} = \left|\Delta\vec{r}_{0,1}\right| + \left|\Delta\vec{r}_{1,2}\right| + \left|\Delta\vec{r}_{2,3}\right| + \ldots + \left|\Delta\vec{r}_{N-1,N}\right|$$

where N is the number of very small displacements. (For this to be exactly true we have to take the limit as N goes to infinity and each displacement magnitude goes to zero.) Now, using ″the shortest distance between two points is a straight line,″ we have

$$\left|\Delta\vec{r}_{0,N}\right| \le \left|\Delta\vec{r}_{0,1}\right| + \left|\Delta\vec{r}_{1,2}\right| + \left|\Delta\vec{r}_{2,3}\right| + \ldots + \left|\Delta\vec{r}_{N-1,N}\right|,$$

where $\left|\Delta\vec{r}_{0,N}\right|$ is the magnitude of the net displacement.

Hence, we have shown that the magnitude of the displacement of a particle is less than or equal to the distance it travels along its path.

6 • Two astronauts are working on the lunar surface to install a new telescope. The acceleration due to gravity on the Moon is only 1.64 m/s^2. One astronaut tosses a wrench to the other astronaut but the speed of throw is excessive and the wrench goes over her colleague's head. When the wrench is at the highest point of its trajectory (*a*) its velocity and acceleration are both zero, (*b*) its velocity is zero but its acceleration is

nonzero, (*c*) its velocity is nonzero but its acceleration is zero, (*d*) its velocity and acceleration are both nonzero,
(*e*) insufficient information is given to choose between any of the previous choices.

Determine the Concept When the wrench reaches its maximum height, it is still moving horizontally but its acceleration is downward. $\boxed{(\boldsymbol{d})}$ is correct.

19 • True or false (Ignore any effects due to air resistance):

(*a*) When a projectile is fired horizontally, it takes the same amount of time to reach the ground as an identical projectile dropped from rest from the same height.
(*b*) When a projectile is fired from a certain height at an upward angle, it takes longer to reach the ground than does an identical projectile dropped from rest from the same height.
(*c*) When a projectile is fired horizontally from a certain height, it has a higher speed upon reaching the ground than does an identical projectile dropped from rest from the same height.

(*a*) True. In the absence of air resistance, both projectiles experience the same downward acceleration. Because both projectiles have initial vertical velocities of zero, their vertical motions must be identical.

(*b*) True. When a projectile is fired from a certain height at an upward angle, its time in the air is twice the time it takes to fall from its maximum height. This distance is greater than it is when the projectile is fired horizontally from the same height.

(*c*) True. When a projectile is fired horizontally, its velocity upon reaching the ground has a horizontal component in addition to the vertical component it has when it is dropped from rest. The magnitude of this velocity is related to its horizontal and vertical components through the Pythagorean Theorem.

23 • Figure 3-29 represents the trajectory of a projectile going from A to E. Air resistance is negligible. What is the direction of the acceleration at point B? (*a*) up and to the right, (*b*) down and to the left, (*c*) straight up, (*d*) straight down, (*e*) The acceleration of the ball is zero.

Determine the Concept $\boxed{(\boldsymbol{d})}$ is correct. In the absence of air resistance, the acceleration of the ball depends only on the *change in its velocity* and is independent of its velocity. As the ball moves along its trajectory between points A and C, the vertical component of its velocity decreases and the *change* in its velocity is a downward pointing vector. Between points C and E, the vertical component of its velocity increases and the *change* in its velocity is also a downward pointing vector. There is no change in the horizontal component of the velocity.

24 • Figure 3-29 represents the trajectory of a projectile going from A to E. Air resistance is negligible. (*a*) At which point(s) is the speed the greatest? (*b*) At which point(s) is the speed the least? (*c*) At which two points is the speed the same? Is the velocity also the same at these points?

Determine the Concept In the absence of air resistance, the horizontal component of the velocity remains constant throughout the flight. The vertical component has its maximum values at launch and impact.

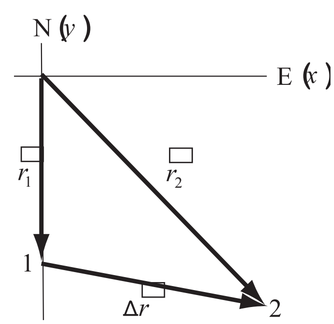
(*a*) The speed is greatest at A and E.

(*b*) The speed is least at point C.

(*c*) The speed is the same at A and E. No. The horizontal components are equal at these points but the vertical components are oppositely directed.

45 • A stationary radar operator determines that a ship is 10 km due south of him. An hour later the same ship is 20 km due southeast. If the ship moved at constant speed and always in the same direction, what was its velocity during this time?

Picture the Problem For constant speed and direction, the instantaneous velocity is identical to the average velocity. Take the origin to be the location of the stationary radar and let the $+x$ direction be to the East and the $+y$ direction be to the North.



Express the average velocity:

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} \qquad (1)$$

Determine the position vectors $\vec{r}_1$ and $\vec{r}_2$:

$$\vec{r}_1 = (-10\,\text{km})\hat{j}$$

and

$$\vec{r}_2 = (14.1\,\text{km})\hat{i} + (-14.1\,\text{km})\hat{j}$$

Find the displacement vector $\Delta\vec{r}$:

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (14.1\,\text{km})\hat{i} + (-4.1\,\text{km})\hat{j}$$

Substitute for $\Delta\vec{r}$ and Δt in equation (1) to find the average velocity.

$$\vec{\mathbf{v}}_{av} = \frac{(14.1\,\text{km})\hat{\mathbf{i}} + (-4.1\,\text{km})\hat{\mathbf{j}}}{1.0\,\text{h}} = \boxed{(14\,\text{km/h})\hat{\mathbf{i}} + (-4.1\,\text{km/h})\hat{\mathbf{j}}}$$

48 • Initially, a swift-moving hawk is moving due west with a speed of 30 m/s; 5.0 s later it is moving due north with a speed of 20 m/s. (*a*) What are the magnitude and direction of $\Delta\vec{v}_{\text{av}}$ during this 5.0-s interval? (*b*) What are the magnitude and direction of $\vec{a}_{\text{av}}$ during this 5.0-s interval?

Picture the Problem Choose a coordinate system in which north coincides with the positive *y* direction and east with the positive *x* direction. Expressing the hawk's velocity vectors is the first step in determining $\Delta\vec{v}$ and $\vec{a}_{\text{av}}$.

(*a*) The change in the hawk's velocity during this interval is:

$$\Delta\vec{v}_{\text{av}} = \vec{v}_{\text{N}} - \vec{v}_{\text{W}}$$

$\vec{v}_{\text{W}}$ and $\vec{v}_{\text{N}}$ are given by:

$$\vec{\mathbf{v}}_{\text{W}} = -(30\text{ m/s})\hat{i} \text{ and } \vec{\mathbf{v}}_{\text{N}} = (20\text{ m/s})\hat{j}$$

Substitute for $\vec{v}_{\text{W}}$ and $\vec{v}_{\text{N}}$ and evaluate $\Delta\vec{v}$:

$$\Delta\vec{v}_{\text{av}} = (20\text{ m/s})\hat{j} - \left[-(30\text{ m/s})\hat{i}\right] = (30\text{ m/s})\hat{i} + (20\text{ m/s})\hat{j}$$

The magnitude of $\Delta\vec{\mathbf{v}}_{\text{av}}$ is given by:

$$\left|\Delta\vec{v}_{\text{av}}\right| = \sqrt{\Delta v_x^2 + \Delta v_y^2}$$

Substitute numerical values and evaluate Δv:

$$\left|\Delta\vec{v}_{\text{av}}\right| = \sqrt{(30\text{ m/s})^2 + (20\text{ m/s})^2} = \boxed{36\text{ m/s}}$$

The direction of Δv is given by:

$$\theta = 180° - \tan^{-1}\left(\frac{\Delta v_y}{\Delta v_x}\right)$$

where θ is measured from the positive *x* axis.

Substitute numerical values and evaluate θ:

$$\theta = 180° - \tan^{-1}\left(\frac{20\text{ m/s}}{30\text{ m/s}}\right) = 180° - 34° = 146° = \boxed{150°}$$

(*b*) The hawk's average acceleration during this interval is:

$$\vec{\mathbf{a}}_{\text{av}} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t}$$

Substitute for $\Delta\vec{v}$ and Δt to obtain:

$$\vec{\mathbf{a}}_{\text{av}} = \frac{(30\text{ m/s})\hat{i} + (20\text{ m/s})\hat{j}}{5.0\text{ s}} = (6.0\text{ m/s}^2)\hat{i} + (4.0\text{ m/s}^2)\hat{j}$$

The magnitude of $\vec{a}_{\text{av}}$ is given by:

$$\left|\vec{a}_{\text{av}}\right| = \sqrt{a_x^2 + a_y^2}$$

Substitute numerical values and evaluate $|\vec{a}_{av}|$:

$$|\vec{a}_{av}| = \sqrt{(6.0\text{ m/s}^2) + (4.0\text{ m/s}^2)}$$
$$= \boxed{7.2\text{ m/s}^2}$$

The direction of $\vec{a}_{av}$ is given by:

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right)$$

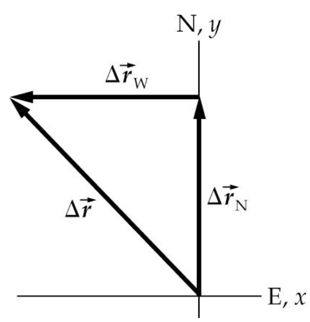
Substitute numerical values and evaluate θ:

$$\theta = \tan^{-1}\left(\frac{4.0\frac{\text{m}}{\text{s}^2}}{6.0\frac{\text{m}}{\text{s}^2}}\right) = \boxed{34°}$$

where θ is measured from the positive x axis.

53 •• **[SSM]** Starting from rest at a dock, a motor boat on a lake heads north while gaining speed at a constant 3.0 m/s^2 for 20 s. The boat then heads west and continues for 10 s at the speed that it had at 20 s. (*a*) What is the average velocity of the boat during the 30-s trip? (*b*) What is the average acceleration of the boat during the 30-s trip? (*c*) What is the displacement of the boat during the 30-s trip?

Picture the Problem The displacements of the boat are shown in the figure. Let the +*x* direction be to the east and the +*y* direction be to the north. We need to determine each of the displacements in order to calculate the average velocity of the boat during the 30-s trip.



(*a*) The average velocity of the boat is given by:

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} \quad (1)$$

The total displacement of the boat is given by:

$$\Delta\vec{r} = \Delta\vec{r}_N + \Delta\vec{r}_W$$
$$= \tfrac{1}{2}a_N(\Delta t_N)^2\hat{j} + v_W\Delta t_W(-\hat{i}) \quad (2)$$

To calculate the displacement we first have to find the speed after the first 20 s:

$$v_W = v_{N,f} = a_N\Delta t_N$$

Substitute numerical values and evaluate v_W:

$$v_W = (3.0\text{ m/s}^2)(20\text{ s}) = 60\text{ m/s}$$

Substitute numerical values in equation (2) and evaluate $\Delta\vec{\mathbf{r}}(30\text{ s})$:

$$\Delta\vec{r}(30\text{ s}) = \tfrac{1}{2}(3.0\text{ m/s}^2)(20\text{ s})^2\,\hat{j} - (60\text{ m/s})(10\text{ s})\,\hat{i} = (600\text{ m})\hat{j} - (600\text{ m})\hat{i}$$

Substitute numerical values in equation (1) to find the boat's average velocity:

$$\vec{\mathbf{v}}_{av} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t} = \frac{(600\text{ m})(-\hat{\mathbf{i}}+\hat{\mathbf{j}})}{30\text{ s}} = \boxed{(20\text{ m/s})(-\hat{\mathbf{i}}+\hat{\mathbf{j}})}$$

(*b*) The average acceleration of the boat is given by:

$$\vec{\mathbf{a}}_{av} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{\Delta t}$$

Substitute numerical values and evaluate $\vec{a}_{av}$:

$$\vec{a}_{av} = \frac{(-60\text{ m/s})\hat{i} - 0}{30\text{ s}} = \boxed{(-2.0\text{ m/s}^2)\hat{i}}$$

(*c*) The displacement of the boat from the dock at the end of the 30-s trip was one of the intermediate results we obtained in Part (*a*).

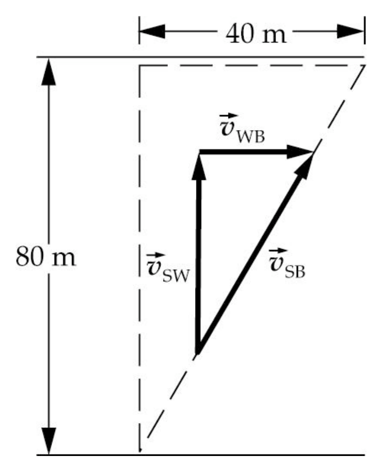
$$\Delta\vec{r} = (600\text{ m})\hat{j} + (-600\text{ m})\hat{i} = \boxed{(600\text{ m})(-\hat{i}+\hat{j})}$$

56 •• A swimmer *heads* directly across a river, swimming at 1.6 m/s relative to the water. She arrives at a point 40 m downstream from the point directly across the river, which is 80 m wide. (*a*) What is the speed of the river current? (*b*) What is the swimmer's speed relative to the shore? (*c*) In what direction should the swimmer head in order to arrive at the point directly opposite her starting point?

Picture the Problem Let $\vec{v}_{SB}$ represent the velocity of the swimmer relative to the shore; $\vec{v}_{SW}$ the velocity of the swimmer relative to the water; and $\vec{v}_{WB}$ the velocity of the water relative to the shore; i.e.,

$$\vec{v}_{SB} = \vec{v}_{SW} + \vec{v}_{WB}$$

The current of the river causes the swimmer to drift downstream.



(*a*) The triangles shown in the figure are similar right triangles. Set up a proportion between their sides and solve for the speed of the water

$$\frac{v_{WB}}{v_{SW}} = \frac{40\text{ m}}{80\text{ m}}$$

and

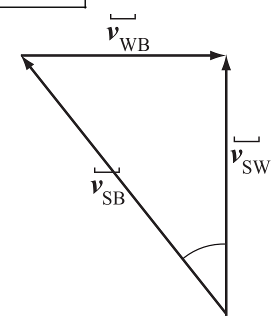
relative to the bank:

$$v_{WB} = \tfrac{1}{2}(1.6\,\text{m/s}) = \boxed{0.80\,\text{m/s}}$$

(*b*) Use the Pythagorean Theorem to solve for the swimmer's speed relative to the shore:

$$v_{SB} = \sqrt{v_{SW}^2 + v_{WB}^2} = \sqrt{(1.6\,\text{m/s})^2 + (0.80\,\text{m/s})^2} = \boxed{1.8\,\text{m/s}}$$

(*c*) The swimmer should head in a direction such that the upstream component of her velocity relative to the shore ($\vec{v}_{SB}$) is equal to the speed of the water relative to the shore ($\vec{v}_{WB}$):



Referring to the diagram, relate $\sin\theta$ to $\vec{v}_{WB}$ and $\vec{v}_{SB}$:

$$\sin\theta = \frac{v_{WB}}{v_{SB}} \Rightarrow \theta = \sin^{-1}\left(\frac{v_{WB}}{v_{SB}}\right)$$

Substitute numerical values and evaluate θ:

$$\theta = \sin^{-1}\left(\frac{0.80\,\text{m/s}}{1.8\,\text{m/s}}\right) = \boxed{27^\circ}$$

62 •• While walking between gates at an airport, you notice a child running along a moving walkway. Estimating that the child runs at a constant speed of 2.5 m/s relative to the surface of the walkway, you decide to try to determine the speed of the walkway itself. You watch the child run on the entire 21-m walkway in one direction, immediately turn around, and run back to his starting point. The entire trip taking a total elapsed time of 22 s. Given this information, what is the speed of the moving walkway relative to the airport terminal?

Picture the Problem We don't need to know which direction the child ran first, and which he ran second. Because we have the total length of the walkway, the elapsed time for the round trip journey, and the child's running speed relative to the walkway, we are given sufficient information to determine the moving walkway's speed. The distance covered in the airport is 21 m, and this is covered in a total time of 22 s. When the child runs in the direction of the walkway, his velocity in the airport is the sum of his walking velocity, $\vec{v}_{child}$, and the walkway velocity, $\vec{v}_{ww}$. On the return trip, the velocity in the airport is the difference of these two velocities.

Express the total time for the child's run in terms of his running times with and against the moving walkway:

$$\Delta t_{tot} = \Delta t_{with} + \Delta t_{against}$$

In terms of the length L of the walkway and the speeds, relative to the airport, of the child running with and against the moving walkway:

$$\Delta t_{\text{tot}} = \frac{L}{v_{\text{with}}} + \frac{L}{v_{\text{against}}} \qquad (1)$$

The speeds of the child relative to the airport, then, in each case, are:

$$\boldsymbol{v}_{\text{with}} = \left|\vec{\boldsymbol{v}}_{\text{child}}\right| + \left|\vec{\boldsymbol{v}}_{\text{ww}}\right| = \boldsymbol{v}_{\text{child}} + \boldsymbol{v}_{\text{ww}}$$

and

$$\boldsymbol{v}_{\text{against}} = \left|\vec{\boldsymbol{v}}_{\text{child}}\right| - \left|\vec{\boldsymbol{v}}_{\text{ww}}\right| = \boldsymbol{v}_{\text{child}} - \boldsymbol{v}_{\text{ww}}$$

Substitute for v_{with} and v_{against} in equation (1) to obtain:

$$\Delta t_{\text{tot}} = \frac{L}{v_{\text{child}} + v_{\text{ww}}} + \frac{L}{v_{\text{child}} - v_{\text{ww}}}$$

Solving this equation for v_{ww} yields: (find common denominator)

$$v_{\text{ww}} = \sqrt{v_{\text{child}}^2 - \frac{2v_{\text{child}}L}{\Delta t_{\text{tot}}}}$$

Substitute numerical values and evaluate v_{ww}:

$$v_{\text{ww}} = \sqrt{(2.5\text{ m/s})^2 - \frac{2(2.5\text{ m/s})(21\text{ m})}{22\text{ s}}} = \boxed{1.2\text{ m/s}}$$

Alternative Interpretation:
If child runs along walkway in one direction and returns by running around then we have two effective speeds for the child:

$$v_{\text{on}} = \left|\vec{\mathbf{v}}_{\text{child}}\right| + \left|\vec{\mathbf{v}}_{\text{ww}}\right| = v_{\text{child}} + v_{\text{ww}} \text{ and } v_{\text{off}} = \left|\vec{\mathbf{v}}_{\text{child}}\right| = v_{\text{child}}$$

In terms of the length L of the walkway and the speeds, relative to the airport, of the child running on and off the moving walkway:

$$\Delta t_{\text{tot}} = \frac{L}{v_{\text{on}}} + \frac{L}{v_{\mathit{off}}}$$

Substitute for v_{on} and v_{off} to obtain:

$$\Delta t_{\text{tot}} = \frac{L}{v_{\text{child}} + v_{\text{ww}}} + \frac{L}{v_{\text{child}}}$$

Solving this equation for v_{ww} yields:

$$v_{\text{ww}} = \frac{2L - v_{\text{child}}\Delta t_{\text{tot}}}{\left(\Delta t_{\text{tot}} - \frac{L}{v_{\text{child}}}\right)}$$

Substitute numerical values and evaluate $v_{\text{ww}} = \boxed{-0.96\text{ m/s}}$

The negative velocity means the walkway is moving in the opposite direction as the child is running (which would be the fun way to run if you were a kid).

69 •• (*a*) What are the period and speed of the motion of a person on a carousel if the person has an acceleration magnitude of 0.80 m/s^2 when she is standing 4.0 m from the axis? (*b*) What are her acceleration magnitude and speed if she then moves in to a distance of 2.0 m from the carousel center and the carousel keeps rotating with the same period?

Picture the Problem The person riding on this carousel experiences a centripetal acceleration due to the fact that her velocity is continuously changing. Use the expression for centripetal acceleration to relate her speed to her centripetal acceleration and the relationship between distance, speed, and time to find the period of her motion.

In general, the acceleration and period of any object moving in a circular path at constant speed are given by:

$$a_c = \frac{v^2}{r} \quad (1)$$

and

$$T = \frac{2\pi r}{v} \quad (2)$$

Solving equation (1) for v yields:

$$v = \sqrt{a_c r} \quad (3)$$

Substituting for v in equation (2) yields:

$$T = \frac{2\pi r}{\sqrt{a_c r}} = 2\pi\sqrt{\frac{r}{a_c}} \quad (4)$$

(*a*) Substitute numerical values in equation (3) and evaluate v for a person standing 4.0 m from the axis:

$$v = \sqrt{(0.80\ \text{m/s}^2)(4.0\ \text{m})} = 1.79\ \text{m/s} = \boxed{1.8\ \text{m/s}}$$

Substitute numerical values in equation (4) and evaluate T for a person standing 4.0 m from the axis:

$$T = 2\pi\sqrt{\frac{4.0\ \text{m}}{0.80\ \text{m/s}^2}} = 14.05\ \text{s} = \boxed{14\ \text{s}}$$

(*b*) Solve equation (2) for v to obtain:

$$v = \frac{2\pi r}{T}$$

For a person standing 2.0 m from the axis:

$$v = \frac{2\pi(2.0\ \text{m})}{14.05\ \text{s}} = 0.894\ \text{m/s} = \boxed{0.89\ \text{m/s}}$$

From equation (1) we have, for her acceleration:

$$a_c = \frac{(0.894\ \text{m/s})^2}{2.0\ \text{m}} = \boxed{0.40\ \text{m/s}^2}$$

72 • While trying out for the position of pitcher on your high school baseball team, you throw a fastball at 87 mi/h toward home plate, which is 18.4 m away. How far does the ball drop due to effects of gravity by the time it reaches home plate? (Ignore any effects due to air resistance.)

Picture the Problem Neglecting air resistance, the accelerations of the ball are constant and the horizontal and vertical motions of the ball are independent of each other. We can use the horizontal motion to determine the time-of-flight and then use this information to determine the distance the ball drops. Choose a coordinate system in which the origin is at the point of release of the ball, downward is the positive y direction, and the horizontal direction is the positive x direction.

Express the vertical displacement of the ball:

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

or, because $v_{0y} = 0$ and $a_y = g$,

$$\Delta y = \tfrac{1}{2}g(\Delta t)^2 \qquad (1)$$

Use $v_x = \Delta x/\Delta t$ to express the time of flight:

$$\Delta t = \frac{\Delta x}{v_x}$$

Substitute for Δt in equation (1) to obtain:

$$\Delta y = \tfrac{1}{2}g\left(\frac{\Delta x}{v_x}\right)^2 = \frac{g(\Delta x)^2}{2(v_x)^2}$$

Substitute numerical values and evaluate Δy:

$$\Delta y = \frac{\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(18.4\ \text{m})^2}{2\left(87\ \frac{\text{mi}}{\text{h}}\cdot\frac{0.4470\ \frac{\text{m}}{\text{s}}}{1\frac{\text{mi}}{\text{h}}}\right)^2} = \boxed{1.1\ \text{m}}$$

75 •• In Figure 3.35, what is the minimum initial speed of the dart if it is to hit the monkey before the monkey hits the ground, which is 11.2 m below the initial position of the monkey, if x is 50 m and $h = 10$ m? You aim directly at the monkey and the monkey drops at the exact same time as the dart is fired. (Ignore any effects due to air resistance.)

Picture the Problem Example 3-12 shows that the dart will hit the monkey unless the dart hits the ground before reaching the monkey's line of fall. What initial speed does the dart need in order to just reach the monkey's line of fall? First, we will calculate the fall time of the monkey, and then we will calculate the horizontal component of the dart's velocity.

Relate the horizontal velocity of the dart to its launch angle and initial velocity v_0:

$$\boldsymbol{v}_x = \boldsymbol{v}_0\cos\boldsymbol{\theta} \Rightarrow \boldsymbol{v}_0 = \frac{\boldsymbol{v}_x}{\cos\boldsymbol{\theta}}$$

Use the definition of v_x and the fact that, in the absence of air resistance, it is constant to obtain:

$$v_x = \frac{\Delta x}{\Delta t}$$

Substituting in the expression for v_0 yields:

$$v_0 = \frac{\Delta x}{(\cos\theta)\Delta t} \qquad (1)$$

Using a constant-acceleration equation, relate the monkey's fall distance to the fall time:

$$\Delta h = \tfrac{1}{2} g(\Delta t)^2 \Rightarrow \Delta t = \sqrt{\frac{2\Delta h}{g}}$$

Substituting for Δt in equation (1) yields:

$$v_0 = \frac{\Delta x}{\cos\theta}\sqrt{\frac{g}{2\Delta h}} \qquad (2)$$

Let θ be the angle the barrel of the dart gun makes with the horizontal. Then:

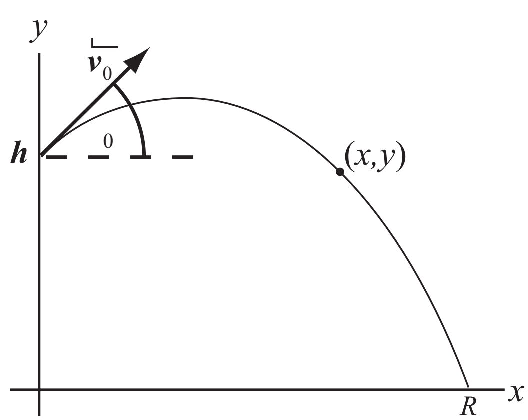
$$\theta = \tan^{-1}\left(\frac{10\,\text{m}}{50\,\text{m}}\right) = 11.3°$$

Substitute numerical values in equation (2) and evaluate v_0:

$$\boldsymbol{v}_0 = \frac{50\,\text{m}}{\cos 11.3°}\sqrt{\frac{9.81\,\text{m/s}^2}{2(11.2\,\text{m})}} = \boxed{34\,\text{m}}$$

88 •• The speed of an arrow fired from a compound bow is about 45.0 m/s. (*a*) A Tartar archer sits astride his horse and launches an arrow into the air, elevating the bow at an angle of 10º above the horizontal. If the arrow is 2.25 m above the ground at launch, what is the arrow's horizontal range? Assume that the ground is level, and ignore any effects due to air resistance. (*b*) Now assume that his horse is at full gallop and moving in the same direction as the direction the archer will fire the arrow. Also assume that the archer elevates the bow at the same elevation angle as in Part (*a*) and fires. If the horse's speed is 12.0 m/s, what is the arrow's horizontal range now?

Picture the Problem Choose a coordinate system in which the origin is at ground level. Let the positive *x* direction be to the right and the positive *y* direction be upward. We can apply constant-acceleration equations to obtain equations in time that relate the range to the initial horizontal speed and the height *h* to which the initial upward speed. Eliminating time from these equations will leave us with a quadratic equation in *R,* the solution to which will give us the range of the arrow. In (*b*), we'll find the launch speed and angle as viewed by an observer who is at rest on the ground and then use these results to find the arrow's range when the horse is moving at 12.0 m/s.



(*a*) Use constant-acceleration equations to express the horizontal and vertical coordinates of the arrow's motion:

$$\Delta x = x - x_0 = v_{0x} t$$

and

$$y = h + v_{0y} t + \tfrac{1}{2}(-g)t^2$$

where

$$\boldsymbol{v}_{0x} = \boldsymbol{v}_0 \cos\boldsymbol{\theta}_0 \text{ and } \boldsymbol{v}_{0y} = \boldsymbol{v}_0 \sin\boldsymbol{\theta}_0$$

Solve the x-component equation for time:

$$t = \frac{\Delta x}{v_{0x}} = \frac{\Delta x}{v_0 \cos\theta_0}$$

Eliminating time from the y-component equation yields:

$$y = h + v_{0y}\frac{\Delta x}{v_{0x}} - \frac{1}{2} g\left(\frac{\Delta x}{v_{0x}}\right)^2$$

When $y = 0$, $\Delta x = R$ and:

$$0 = \boldsymbol{h} + (\tan\boldsymbol{\theta}_0)\boldsymbol{R} - \frac{\boldsymbol{g}}{2\boldsymbol{v}_0^2 \cos^2\boldsymbol{\theta}_0}\boldsymbol{R}^2$$

Solve for the range R to obtain:

$$\boldsymbol{R} = \frac{\boldsymbol{v}_0^2}{2\boldsymbol{g}}\sin 2\boldsymbol{\theta}_0\left(1 + \sqrt{1 + \frac{2\boldsymbol{gh}}{\boldsymbol{v}_0^2 \sin^2\boldsymbol{\theta}_0}}\right)$$

Substitute numerical values and evaluate R:

$$\boldsymbol{R} = \frac{(45.0\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)}\sin 20°\left(1 + \sqrt{1 + \frac{2(9.81\,\text{m/s}^2)(2.25\,\text{m})}{(45.0\,\text{m/s})^2(\sin^2 10°)}}\right) = \boxed{82\,\text{m}}$$

(*b*) Express the speed of the arrow in the horizontal direction:

$$\begin{aligned}\boldsymbol{v}_x &= \boldsymbol{v}_{\text{arrow}} + \boldsymbol{v}_{\text{archer}} \\ &= (45.0\,\text{m/s})\cos 10° + 12.0\,\text{m/s} \\ &= 56.32\,\text{m/s}\end{aligned}$$

Express the vertical speed of the arrow:

$$\boldsymbol{v}_y = (45.0\,\text{m/s})\sin 10° = 7.814\,\text{m/s}$$

Express the angle of elevation from the perspective of someone on the ground:

$$\boldsymbol{\theta}_0 = \tan^{-1}\left(\frac{\boldsymbol{v}_y}{\boldsymbol{v}_x}\right)$$

Substitute numerical values and evaluate θ_0 :

$$\boldsymbol{\theta}_0 = \tan^{-1}\left(\frac{7.814\,\text{m/s}}{56.32\,\text{m/s}}\right) = 7.899°$$

The arrow's speed relative to the ground is given by:

$$\boldsymbol{v}_0 = \sqrt{\boldsymbol{v}_x^2 + \boldsymbol{v}_y^2}$$

Substitute numerical values and evaluate v_0:

$$v_0 = \sqrt{(56.32\,\text{m/s})^2 + (7.814\,\text{m/s})^2} = 56.86\,\text{m/s}$$

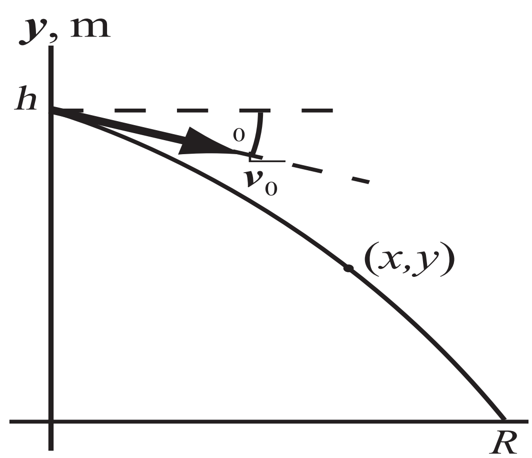
Substitute numerical values and evaluate R:

$$R = \frac{(56.86\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)}\sin(15.8°)\left(1+\sqrt{1+\frac{2(9.81\,\text{m/s}^2)(2.25\,\text{m})}{(56.86\,\text{m/s})^2(\sin^2 7.899°)}}\right) = \boxed{0.10\,\text{km}}$$

Remarks: An alternative solution for part (*b*) is to solve for the range in the reference frame of the archer and then add to it the distance the frame travels, relative to the earth, during the time of flight.

106 •• During a do-it-yourself roof repair project, you are on the roof of your house and accidentally drop your hammer. The hammer then slides down the roof at constant speed of 4.0 m/s. The roof makes an angle of 30º with the horizontal, and its lowest point is 10 m from the ground. (*a*) How long after leaving the roof does the hammer hit the ground? (*b*) What is the horizontal distance traveled by the hammer between the instant it leaves the roof and the instant it hits the ground? (Ignore any effects due to air resistance.)

Picture the Problem In the absence of air resistance, the hammer experiences constant acceleration as it falls. Choose a coordinate system with the origin and coordinate axes as shown in the figure and use constant-acceleration equations to describe the x and y coordinates of the hammer along its trajectory. We'll use the equation describing the vertical motion to find the time of flight of the hammer and the equation describing the horizontal motion to determine its range.



(*a*) Using a constant-acceleration equation, express the x coordinate of the hammer as a function of time:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

or, because $x_0 = 0$, $v_{0x} = v_0\cos\theta_0$, and $a_x = 0$,

$$x = (v_0\cos\theta_0)t$$

Using a constant-acceleration equation, express the y coordinate of the hammer as a function of time:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

or, because $y_0 = h$, $v_{0y} = v_0\sin\theta$, and $a_y = -g$,

$$y = h + (v_0\sin\theta)t - \tfrac{1}{2}gt^2$$

Substituting numerical values yields:

$$y = 10\,\text{m} + (-4.0\,\text{m/s})(\sin 30°)\,t - \tfrac{1}{2}(9.81\,\text{m/s}^2)\,t^2$$

When the hammer hits the ground, $y = 0$ and our equation becomes:

$$0 = 10\,\text{m} - (4.0\ \text{m/s})\sin 30°\,t_{\text{fall}} - \tfrac{1}{2}(9.81\,\text{m/s}^2)\,t_{\text{fall}}^2$$

Use the quadratic formula or your graphing calculator to solve for the time of fall:

$$t_{\text{fall}} = 1.238\,\text{s} = \boxed{1.2\,\text{s}}$$

(*b*) Use the *x*-coordinate equation to express the horizontal distance traveled by the hammer as a function of its time-of-fall:

$$R = v_{0x} t_{\text{fall}} = (v_0 \cos\theta_0)\, t_{\text{fall}}$$

Substitute numerical values and evaluate *R*:

$$\boldsymbol{R} = (4.0\,\text{m/s})(\cos 30°)(1.238\,\text{s}) = \boxed{4.3\,\text{m}}$$

111 •• **[SSM]** Plane A is headed due east, flying at an air speed of 400 mph. Directly below, at a distance of 4000 ft, plane B is headed due north, flying at an air speed of 700 mph. Find the velocity vector of plane B relative to A.

Picture the Problem The velocity of plane B relative to plane A is independent of the reference frame in which the calculation is done. The solution that follows uses the reference frame of the air. Choose a coordinate system in which the +*x* axis is to the east and the +*y* axis is to the north and write the velocity vectors for the two airplanes using unit vector notation. We can then use this vector to express the relative velocity in speed and heading form.

The velocity of plane B relative to plane A is given by:

$$\vec{\mathbf{v}}_{\text{BA}} = \vec{\mathbf{v}}_{\text{Ba}} + \vec{\mathbf{v}}_{\text{aA}} \qquad (1)$$

where the subscript a refers to the air.

Using unit vector notation, write expressions for $\vec{\mathbf{v}}_{\text{Ba}}$ and $\vec{\mathbf{v}}_{\text{Aa}}$:
Note: $\vec{\mathbf{v}}_{\text{Aa}} = -\vec{\mathbf{v}}_{\text{aA}}$

$$\vec{\boldsymbol{v}}_{\text{Ba}} = (700\,\text{mi/h})\,\hat{\boldsymbol{j}}$$

and

$$\vec{\boldsymbol{v}}_{\text{Aa}} = (400\,\text{mi/h})\,\hat{\boldsymbol{i}}$$

Substitute for $\vec{\boldsymbol{v}}_{\text{Ba}}$ and $\vec{\boldsymbol{v}}_{\text{Aa}}$ in equation (1) to obtain:

$$\boxed{\begin{aligned}\vec{\mathbf{v}}_{\text{BA}} &= (700\ \text{mi/h})\,\hat{j} - (400\ \text{mi/h})\,\hat{i} \\ &= -(400\ \text{mi/h})\,\hat{i} + (700\ \text{mi/h})\,\hat{j}\end{aligned}}$$

The speed of plane B relative to plane is the magnitude of $\vec{v}_{BA}$:

$$|\vec{v}_{BA}| = \sqrt{(-400\text{ mi/h})^2 + (700\text{ mi/h})^2}$$
$$= \boxed{806\text{ mi/h}}$$

The heading of plane B relative to plane A is:

$$\theta_{BA} = \tan^{-1}\left(\frac{700\text{ mi/h}}{-400\text{ mi/h}}\right)$$
$$= \boxed{60.3^\circ\text{ north of west}}$$